Worksheet 9

1. Show that there exists a pair of people at a party who have the same number of friends in attendance.

    - People at the party have at least one friend
    - Nobody is friend with themselves
    - There are no unrequited friendships

2. Let $S = \{(a, b) \in \mathbb{R} \times \mathbb{R} | 0 \leq a \leq 1, 0 \leq b \leq 1\}$.

   Prove: If $s_1, s_2, s_3, s_4, s_5 \in S$ are distinct, then there exist $1 \leq i \neq j \leq 5$ such that the distance between s_i and s_j is less than or equal to $\frac{\sqrt{2}}{2}$.

3. Prove that $\mathbb{N}$ is infinite.

4. Statement 6.30: Suppose A, B, and C are sets such that $|A| = |B|$ and $|B| = |C|$. Then

   $|A| = |C|$

5. Prove that $\mathbb{Z}$ and $\mathbb{N}$ have the same cardinality.

6. Prove that $\mathbb{R}$ and $(0, 1)$ have the same cardinality.